We have written the electric field for a plane wave as
\[ \vec{E} = E_0 e^{i(kx - \omega t)} \]
or
\[ \vec{E} = E_0 \sin(kx - \omega t) \]
so far, we have ignored the fact that \( E_0 \) is a vector's magnitude and direction.

The first effect of polarization we will consider is reflection from a boundary.

\( S \)-polarization \( \Rightarrow \vec{E} \) field polarized \( \perp \) to plane of incidence

\( P \)-polarization \( \Rightarrow \vec{E} \) field \( \parallel \) parallel to plane of incidence
The punch line: "Fresnel Equations"
Where do these come from?

Maxwell's Equations + Boundary conditions

\[ \oint \vec{E} \cdot d\vec{l} = \frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{s} \]

\[ \oint \vec{E} \cdot d\vec{l} = E_{\parallel} l_1 + E_{\perp} \frac{l_2}{2} - E_{\parallel} \frac{l_2}{2} - E_{\perp} l_1 + E_{\perp} \frac{l_2}{2} - E_{\parallel} \frac{l_2}{2} \]

\[ \oint \vec{E} \cdot d\vec{l} = (E_{\parallel} - E_{\perp}) l_1 = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{s} \]

\[ (E_{\parallel} - E_{\perp}) l_1 = -\frac{\partial}{\partial t} \left( B_1 l_1 + B_2 \frac{l_2}{2} \right) \]

let \( l_2 \) (with or rectangle) → 0
\[
\frac{\partial}{\partial t} (\mathbf{B} \cdot d\mathbf{A}) \to 0
\]

\[
\Rightarrow (E_{1} - E_{3}) \Theta = 0 \quad \text{tangential components of } \mathbf{E} \text{ field continuous.}
\]

It did similar analysis to with Gauss's Law:
\[
\oint \mathbf{E} \cdot d\mathbf{s} = Q_{\text{enc}}
\]

get
\[
\epsilon_{1} E_{1} - \epsilon_{2} E_{2} = 0 \quad \epsilon = n^2
\]

normal components disjointed by

index of refraction \(\leq\) Snell's law

Reflection of Light from boundary depends on index of refraction difference.

Assume non-magnetic material so \(\mu_{i} = \mu_{r} = \mu_{o}\)

\[
\Gamma = \frac{n_{i} \cos \Theta_{i} - n_{r} \cos \Theta_{r}}{n_{i} \cos \Theta_{i} + n_{r} \cos \Theta_{r}}
\]
If \( n_i = n_e \) \( \Rightarrow \) by Snell's law \( n_i \sin \theta_i = n_e \sin \theta_e \)

\( \theta_i = \theta_e \)

\( \Rightarrow \) \( \Gamma_\perp = 0 \) for \( n_i = n_e \)

No reflection unless there is an index mismatch.

What about phase changes upon reflection?

\[
\Gamma_\perp = \frac{n_i \cos \theta_i - n_e \cos \theta_e}{n_i \cos \theta_i + n_e \cos \theta_e} \quad \text{sign of numerator determine } \gamma
\]

\( \Gamma_\perp > 0 \Leftrightarrow \Gamma_\perp < 0 \)

180° phase flip if

\[
n_i \cos \theta_i - n_e \cos \theta_e =
\]

\[
n_i (1 - \sin^2 \theta_i)^{\frac{1}{2}} - n_e (1 - \sin^2 \theta_e)^{\frac{1}{2}}
\]

\[
= n_i (1 - \sin^2 \theta_i)^{\frac{1}{2}} - n_e (1 - \frac{n_i^2 \sin \theta_i}{n_e^2})^{\frac{1}{2}}
\]

\[
= \left( n_i^2 - n_i^2 \sin^2 \theta_i \right)^{\frac{1}{2}} - \left( n_e^2 - n_i^2 \sin^2 \theta_i \right)^{\frac{1}{2}}
\]

\( \Rightarrow \) if \( n_i > n_e \) \( \Rightarrow \) numerator < 0 \( \Rightarrow \) phase flip

\( n_e < n_i \) \( \Rightarrow \) numerator > 0 \( \Rightarrow \) no phase flip
Show animation of Reflection from boundary

Let's plot the Fresnel equations as a function of angle.

(Show Mathcad sheet of Fresnel plots)

Point out Brewster's Angle

\[ \tan \Theta_B = \frac{n_e}{n_i} \]

angle at which reflection of parallel component vanishes.

Point about total Internal Reflection

Chapter 8
Chapter 8 - PART 1

\( \vec{E} \) of a light wave is a vector. The direction of \( \vec{E} \) determines its polarization.

So far, we have been working with linearly polarized light.

Example

\[ \vec{E} = E_0 \hat{x} e^{i(kz - \omega t)} \]

or

\[ \vec{E} = E_0 \hat{x} \cos(kt - \omega t) \]

The above example has \( E \) field polarized in the \( \hat{x} \) direction, with wave propagation in \( z \) direction.

If light is polarized in \( \hat{y} \) direction,

\[ \vec{E} = E_0 \hat{y} \sin(kt - \omega t) \]

Light can also be polarized linearly in the \( x-y \) plane.
Why is polarization important?

Example: interference

\[ \vec{E}_T = \vec{E}_1 + \vec{E}_2 \]

\[ |E_T|^2 = (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2) \]

\[ |E_T|^2 = E_1^2 + E_2^2 + 2 \vec{E}_1 \cdot \vec{E}_2 \]

Interference term.

Interference is zero if \( \vec{E}_1 \) and \( \vec{E}_2 \) are orthogonal.
Circular Polarization

Consider two orthogonal waves

\[ \overrightarrow{E_1} = \overrightarrow{E_0} \times \cos(\frac{k_x}{\epsilon_o} - \omega t) \]
\[ \overrightarrow{E_2} = \overrightarrow{E_0} \times \sin(\frac{k_y}{\epsilon_o} - \omega t) \]

One can describe \( \overrightarrow{E_T} = \overrightarrow{E_1} + \overrightarrow{E_2} \) either

as two linearly polarized waves with a \( \pi/2 \) phase shift.

Or describe it as circularly polarized light

[Diagram of circularly polarized light]

Tip of \( \overrightarrow{E} \) field traces out a circle as the wave propagates

Use Right hand rule to define

Left + Right hand polarization

[Diagram showing right-handed and left-handed polarizations]

(Show Figures of E field of Linear/Circular Light)
\[ \mathbf{E}_R = \mathbf{E}_0 \left( \hat{x} \cos(kz - \omega t) + \hat{y} \sin(kx - \omega t) \right) \]

for left handed

in complex notation

\[ \mathbf{E}_R = \frac{\mathbf{E}_0}{\sqrt{2}} (\hat{x} + i \hat{y}) e^{ikz - i\omega t} \]

\[ i = \sqrt{-1} \text{ gives } 90^\circ \text{ phase shift.} \]

\[ \text{In General, light is elliptically polarized} \]

\[ \text{path traced out} \]

by \( \mathbf{E} \) field vector

\[ \text{Polarizer} \Rightarrow \text{transmits light with only one polarization state} \Rightarrow \text{eg. Linear polarizing filters} \]

\[ \text{transmission axis} \]

\( \text{(pass out polarizing filter)} \)
Law of Malus: \[ E_t = E_0 \cos \theta \]

If \( E_0 \perp \) transmission axis

\( \Rightarrow \) all light \( \perp \) blocked

\[ P_t = P_0 \cos^2 \theta \quad \text{Since } P \sim E^2 \]

Lab 3: PART 1 \( \Rightarrow \) verify Law of Malus.

Part 2 \( \Rightarrow \) study properties of materials that affect polarization of light

\( \Rightarrow \) BME section \( \Rightarrow \) d-glucose

\( \Rightarrow \) other section \( \Rightarrow \) wave plates (birefringent crystals)
P1: rotate for maximum transmission

P1 then defines polarization state

keep P1 fixed

Then rotate P2. \( P = P_0 \cos^2 \theta \)

angle between P1 and P2

as P1 fixed and P2 rotates.
4.42] Light is \( \perp \) to glass \((n = 1.522)\).

Determine reflectance \( r \) and transmittance \( t \).

Solution: For \( \perp \) incidence, \( \theta_i = 0 \).

From Snell's Law \( n_1 \sin \theta_i = n_2 \sin \theta_e \).

\[ \Rightarrow \theta_e = 0 \]

Also, since \( \theta_e = 0 \), \( \parallel \) and \( \perp \) should give same answer.

\[ E_\parallel \mid \quad \text{Eq.} 4.432 \quad \Gamma_\parallel = \left( \frac{E_\parallel}{E_\perp} \right) = \frac{n_\parallel \cos \theta_i - n_\perp \cos \theta_e}{n_\parallel \cos \theta_i + n_\perp \sin \theta_i \cos \theta_e} \]

\[ M_\parallel = M_\perp = M. \]

\[ \Gamma_\parallel = 1 - \frac{1.522}{1 + 1.522} = -\frac{.522}{2.522} = -0.207 \]

Note \[ \begin{bmatrix} \Gamma_\parallel \end{bmatrix} \bigg|_{\theta_i = 0} = \begin{bmatrix} \Gamma_{11} \end{bmatrix} \bigg|_{\theta_i = 0} \]

Because of initial direction assumed for \( E \).
\[
\begin{align*}
\tan \theta &= \frac{2 \pi \epsilon_0 \epsilon_r}{\mu_0 \mu_r} \\
&= \frac{2 \pi}{\mu_r + \mu_0} \\
&= \frac{2 \pi}{1.322 + 1} \\
&= 0.793
\end{align*}
\]

Note \( t_\perp + (-t_\parallel) = 1 \) \( 0.793 + 0.207 = 1 \)

Generally true that \( t_\perp + (-t_\parallel) = 1 \)

However \( t_\parallel + t_\perp = 1 \) only at normal incidence.

However \( T + R = 1 \) if no absorption or scattering

\[ \text{Power, not } E \text{ field} \]

8.4 Write expression for P-state lightwave of angular frequency \( \omega \) and amplitude \( E_0 \) propagating along \( x \)-axis with its plane of vibration 25° relative to the \( xy \) plane. \( E(x=0, t=0) = 0 \).

Solution: P-state \( \Rightarrow \) linearly polarized.

\[ \vec{E}(x,t) = E_0 \sin(kx - \omega t) \]

\[ \text{propagation along } x \text{ axis,} \]
\[ \vec{E}_0 = (\cos 25^\circ) \hat{y} + \sin 25^\circ \hat{k} \]

\[ \vec{E}(x,t) = E_0 (\cos 25^\circ \hat{y} + \sin 25^\circ \hat{k}) \sin(kx - wt) \]

**Note:** \[ E(x=0, t=0) = 0. \]

\[ \text{incident light "natural light" is unpolarized} \]

Since initial light unpolarized, you loose \( \frac{1}{2} \) the light through first polarizer.

After first polarizer \[ P = \frac{P_0}{2} \]

After 2nd polarizer, \( \Rightarrow \) law of malus

\[ P = \frac{P_0}{2} \cos^2 \theta \] law of malus.

\( \Rightarrow \) first polarizer \[ P = \frac{(400 \text{ W/m}^2 \cos 40^\circ)^2}{\text{Area}} \]

Since light unpolarized \[ \frac{P}{\text{Area}} = 117.4 \text{ W/m}^2 \]
If incident light is natural light (unpolarized), compute degree of polarization.

\[
\text{degree of polarization} = \frac{I_{\perp}}{I_{\parallel} + I_{\perp}} = \frac{1/2}{1 + 1/2}
\]

\[
\equiv V = \frac{I_{\perp}}{I_{\parallel} + I_{\perp}} \quad \text{Polarized}
\]

\[
\equiv V = \frac{I_{\parallel}}{I_{\parallel} + I_{\perp}} \quad \text{Natural}
\]

Note initial \( I_{\parallel} = 0 \), no degree of polarization.

Use power reflection coefficient.

\[
P \downarrow = \frac{P_0}{2}
\]

\[
P_{\parallel} = \frac{P_0}{2} \quad \text{Initial power in } \parallel\text{ polarized.}
\]

\[
P_{\perp} = \frac{P_0}{2} \quad \text{Initial power in } \perp\text{ polarized.}
\]

\[
R_{\perp} = \frac{\sin^2(\Theta_i - \Theta_e)}{\sin^2(\Theta_i + \Theta_e)}
\]
First determine \( \Theta_e \)

\[
\eta L \sin \Theta_e = \eta e \sin \Theta_e \]

\[
\sin \Theta_e = \frac{1 \sin 40^\circ}{1.5} \Rightarrow \Theta_e = 25.37^\circ 
\]

\[
R_L = \frac{\sin^2 (40 - 25.37)}{\sin^2 (40 + 25.37)} = \frac{0.826(0.638)}{(0.826)} 
\]

\[
R_L = 0.0772 
\]

\[
R_{||} = \frac{\tan^2 (\Theta_e - \Theta_e)}{\tan^2 (\Theta_e + \Theta_e)} = \frac{0.0681}{0.0143} = 4.75 
\]

**Note:** Equal parts of \( \parallel \) and \( \perp \) polarization correspond to unpolarized natural light.

\[
S_0 = I_e = \frac{P_o}{2} (0.0772 - 0.0143) 
\]

\[
I_n = \frac{P_o (0.0143) + \frac{P_o (0.0143)}{2}}{2} \Rightarrow \parallel \text{ contribution} + \perp \text{ contribution} 
\]

\[
V = \frac{I_e}{I_e + I_n} = \frac{(0.0772 - 0.0143) \frac{P_o}{2}}{\frac{P_o (0.0772 - 0.0143) + \frac{P_o (0.0143)}{2}}{2}} 
\]

\[
= \frac{0.629}{0.915} = 0.687 \Rightarrow 68.7\% \text{ polarized} 
\]